Macroscopic Relations in Rarefied Shear Flows

Alexander I. Erofeev, Oscar G. Friedlander

Central Aerohydrodynamic Institute (TsAGI)
1, Zhukovsky str., 140180 Zhukovsky, Russia

Abstract. The plane Couette flow and hypersonic flow past a flat plate at a zero angle of attack are investigated. The relations between stresses and heat fluxes in the Couette flows and the gradients of velocity and temperature are derived. These relations are the generalization of Newton-Fourier (Navier-Stokes) relations for the shear flow with strong nonequilibrium. The relation between longitudinal heat flux, on the one hand, and both transverse heat flux and shear stress with another, is detected. The possibility of application of the relations, derived for Couette flow, to hypersonic flow past flat plate at zero angle of attack is investigated. The limits of applicability of Burnett relations for this flow are determined

INTRODUCTION.

The recent years have been observing the considerable number of attempts to construct macroscopic equations to describe gas flows at not very small values of the local Knudsen number. The example of such flows is the hypersonic flow past thin bodies under small angle of attack. These problems have been studied by Eu B.K. and Cheng H.K in the well-known papers, as well as in numerous papers using the Burnett equations. Besides the general approach, the intense studies of the plane Couette problem were carried out. In paper [1] the structure of asymptotic series of Hilbert normal solution for shear flow corresponding to the plane Couette problem was obtained. There, the one-component gas of Maxwellian molecules was considered. It was shown that the terms of asymptotic series $p_{xy}^{(k)} / p_{xy}^{N}$ and $q_{y}^{(k)} / q_{y}^{F}$ depend only on ratio $a = p_{xy}^{N} / p$. Here, index k is the order of the term, and p_{xy}^{N}, q_{y}^{F} are stress and heat flux in the Navier-Stokes approximation defined by Newton-Fourier relations, p is a pressure. Paper [2] develops these studies. It was shown (with the help of BGK kinetic equation) that the asymptotic series of the normal solution of the kinetic equation can be related to the functions defined at any values of parameter a . Moreover, it was proved that these functions are the solution of the model kinetic equation at zero temperature of plates in the Couette problem. These functions were calculated for the values of parameter a (i.e., the local value of Knudsen number) smaller than 1. In our research these functions are calculated for the kinetic Boltzmann equation. For this purpose the Couette problem with the finite values of wall temperature (i.e., the non-zero thickness of Knudsen layer) is solved. The results obtained in [2] and in the present research have proved that it is impossible to apply both Newton-Fourier relations and Burnett and superburnett relations for the description of strong non-linear shear flows. The review of application of these equations for the Couette problem and other problems with large values of the local Knudsen number can be found in [3-5].

The relations defined in the Couette problem turned out to be useful for the analysis of a more complicated problem. The present research investigates the limits of applicability of Fourier-Newton relations, Burnett relations and relations defined in the Couette problem using the hypersonic flow past a flat plate at zero angle of attack.

The special attention appeals to existing longitudinal heat flux. The value of this heat flux in a studied range of flow parameters (small value of Knudsen number) has the same order, as the normal to plate heat flux. It is shown, that the first term of a Hilbert normal series (the Navier-Stokes approximation) does not account for this effect. It is shown also, that the second term of this series (the Burnett approximation) can qualitatively describe the change of longitudinal heat flux in a gas volume. It concerns only to a moderate non-equilibrium flow (the value of nonequilibrium parameter is $a \le 0.2$).

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14. ABSTRACT
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COUETTE PROBLEM

Problem statement.

The one-component monatomic gas flow was investigated by DSMC method. The supersonic and hypersonic flows between parallel flat plates moving with relative dimensionless velocity S_w and temperature ratio $t_{21} = T_2 / T_1$ were studied. The hard sphere molecules (HS), Maxwellian molecules (MM) and VHS model for Maxwellian molecules (PMM) were chosen as molecular models. To describe the interaction of the molecules with the plates the complete diffusion reflection law was applied. The main goal of this part of the research was to study the possible application of the results of [1, 2] for the gas flows described by Boltzmann equation. Also the diagonal components of stress tensor p_{xx} and p_{yy} and longitudinal heat flux q_x were studied. The latter was detected in [6] where the model BGK kinetic equation was employed.

The DSMC method with majorant collision frequency was used. Time step, size of cells, number of molecules in the cells and sample size were chosen to reduce the statistic error of calculation of hydrodynamic parameters (temperature and velocity) to 0.1 %. When calculating Newton stress p_{xy}^N and Fourier heat flux q_y^F outside the Knudsen layer the linear dependence of the velocity against coordinate s was used. Coordinate s was introduced by a well-known manner [2]: $\mu(T)\dfrac{\partial}{\partial y} = \dfrac{\partial}{\partial s}$, where $\mu(T)$ is a viscosity coefficient. The following dimensionless variables are introduced: coordinate y is referred to the distance between the plates (L), coordinate s is normalized so that its range is within (0, 1), density is referred to average density ρ_{av} introduced as usual, temperature to T_1, velocity to $c_1 = \sqrt{2RT_1}$, pressure and stress to $(1/2)\rho_{av}c_1^2$, heat flux to $(1/2)\rho_{av}c_1^3$. Knudsen number was defined with the help of the distance between the plates and mean free path $\lambda_1 = \dfrac{16}{5\pi}\dfrac{\mu_1}{\rho_{av}}\sqrt{\dfrac{2}{\pi RT_1}}(1-\dfrac{2}{3k})(1-\dfrac{1}{k})$: $Kn = \dfrac{\lambda_1}{L}$.

Here k is a power in the potential of intermolecular interaction $U(r) = cr^{-k}$: k=4 – Maxwellian molecules, $k=\infty$ - molecules – hard sphere.

Results.

First of all, it was found out that the deviation of MM calculation from VHS (PMM) calculation does not exceed 1%. Therefore, the PMM model was used for further investigations. Parameter $a = \left| p_{xy}^N / p \right|$ related to the local value of Knudsen number was naturally chosen as the parameter defining the deviation from equilibrium. Fig.1a shows the dependence of the ratio of real stress to Newton stress p_{xy} / p_{xy}^N against parameter a^2. Both values are constant outside the Knudsen layers. Pressure is also constant (for HS model it is approximately constant). Thus, the solution of problem for fixed boundary conditions is presented by one point in Fig. 1.

Obviously, the normal Hilbert solution exists and can be found even at non-equilibrium parameter larger than 1. This solution of Boltzmann equation for stress is weakly dependent on the molecule models and practically is not different from the solution of BGK equation. At moderately large values of nonequilibrium ($a \le 0.3$) the difference of stress from its Newton approximation is less that 20%. As it is obvious from Fig.1b the mathematical model (kinetic equation) has greater effect on heat flux than stress. The difference of heat flux from Fourier approximation is also larger. Here, we should point out that Burnett approximation gives correction neither to Newton shear stress p_{xy} nor to Fourier heat flux q_y. Besides the transverse heat flux q_y and stress p_{xy} we studied longitudinal heat flux q_x and diagonal components of stress tensor p_{xx} and p_{yy}. In Navier-Stokes approximation these values are equal to zero. In Burnett approximation they are different from zero. Transforming Burnett approximation and neglecting the terms of order $O(Kn^2)$, we can derive the following relations:

$$q_x^B = C q_y^F \frac{p_{xy}^N}{p},\ C_{MM} = 3.500,\ C_{HS} = 3.193$$

$$p_{xx}^B = A_x \frac{(p_{xy}^N)^2}{p} + B_x \frac{(q_y^F)^2}{p2RT},\ p_{yy}^B = A_y \frac{(p_{xy}^N)^2}{p} + B_y \frac{(q_y^F)^2}{p2RT} \tag{1}$$

$$A_{xMM} = 1.600,\ A_{yMM} = -1.200,\ B_{xMM} = B_{yMM} = 0$$

$$A_{xHS} = 1.51,\ B_{xHS} = 0.04695,\ A_{yHS} = -1.163,\ B_{yHS} = -0.0939$$

In relations (1) Burnett diagonal stresses and longitudinal heat flux are expressed through the velocity and temperature gradients as usual. Below these value shall be designated by index B-NS. If in the right side of Eq.(1) we replace p_{xy}^N and q_y^F by p_{xy} and q_y obtained from the Boltzmann solution, then, the new relations shall be designated by additional index B-MC. In this case the dependence of the defined values ($q_x^{B-MC}, p_{xx}^{B-MC}, p_{yy}^{B-MC}$) from the gradients of the hydrodynamic parameters is no longer the second order polynomials. It occurs because values p_{xy} and q_y have been defined by non-linear functions as shown in Fig. 1.

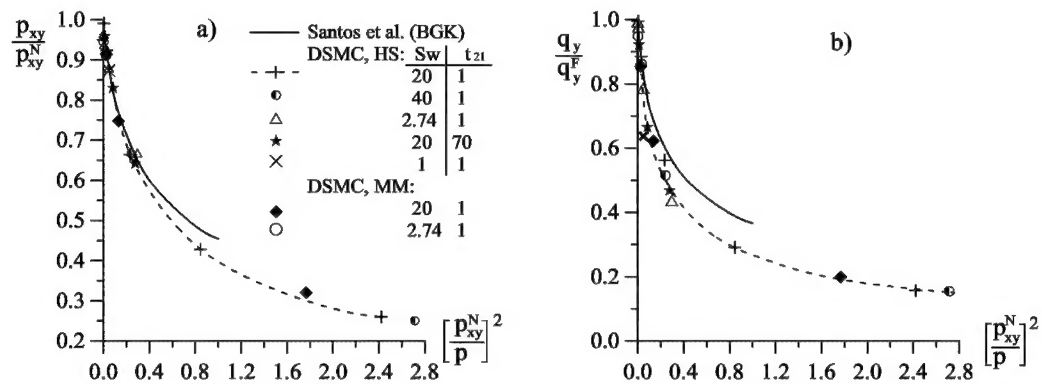

FIGURE 1. Generalized Newton relation for stress (a) and generalized Fourier relation for heat flux (b) vs. nonequilibrium parameter.

Figures 2, 3 present dependencies $q_x(s)$ and $p_{ii}(s)$ for the two flows with Maxwellian molecules and parameters: figure 2 - S_w = 2.379, Kn = 0.032, $t_{21} = 1$; figure 3 - $S_w = 20$, Kn=0.01, $t_{21} = 1$. The results for supersonic flow presented in Fig. 2 prove that Burnett approximation in its classical form (1) gives good qualitative description but leads to quantitative errors of 20 - 30 %. It is interesting to note that approximation B-MC does not give any significant errors in heat flux or normal stresses. In this flow the value of non-equilibrium parameter is $a = 0.13$.

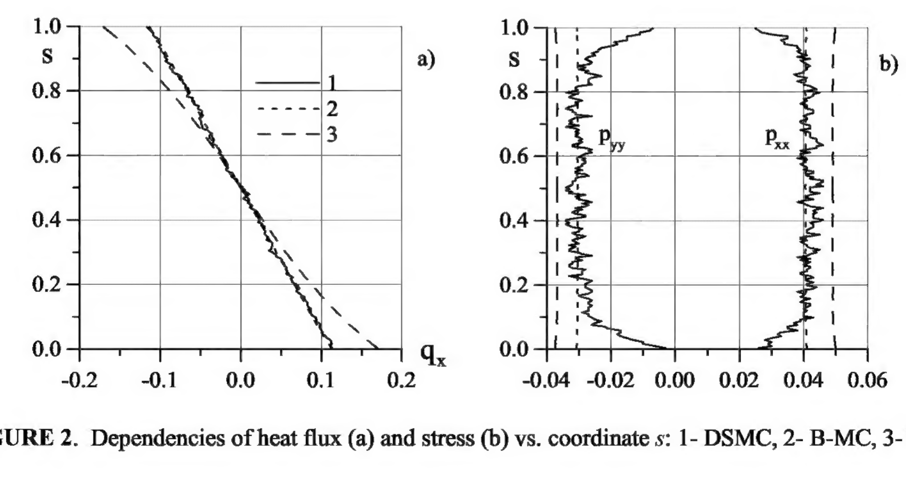

FIGURE 2. Dependencies of heat flux (a) and stress (b) vs. coordinate s: 1- DSMC, 2- B-MC, 3- B-NS.

In hypersonic flow this value is $a = 0.27$. Respectively, the Burnett (B-NS) value of heat flux q_x^{B-NS} is more than twice larger than its real value. And the Burnett value of stresses is more than 1.5 times larger than the real ones. The proposed modification (B-MC) reduces the approximation error, but does not produce the complete solution of the problem. So, the example with the Couette problem proves the real usefulness of the Burnett approximation for weak non-equilibrium flows description, but this approximation is not enough to define the normal solutions of Boltzmann equation for strongly non-equilibrium flows. To derive macroscopic equations, which can describe the tangential heat flux and normal stresses, we should employ the dependencies, which generalize the data presented in (1) and in Fig.1.

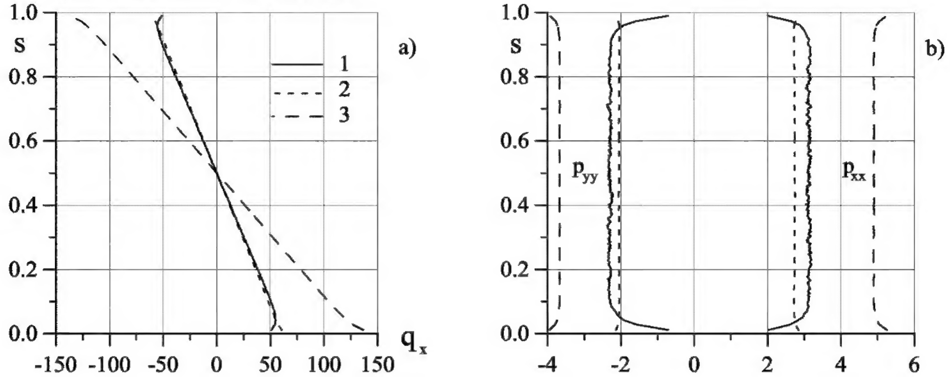

FIGURE 3. Dependencies heat flux (a) and stress (b) on coordinate s: 1- DSMC, 2- B-MC, 3- B-NS.

FLOW PAST A FLAT PLATE

Hypersonic flow past a flat plate at a zero angle of attack is a typical example of the flow close to purely shear one. A lot of papers have earlier shown (for example, see [7]) that at sufficiently small Kn numbers the surface values of shear stress and heat flux normal to the plate can be satisfactorily described by Navier-Stokes equations (approximations) with slip boundary conditions. It is absolutely different with the dissipative quantities out of plate surface. We have considered the hypersonic flow ($S_\infty = 21$) past a plate with constant surface temperature T_w ($t_w = T_w / T_0 = 0.2$, T_0 is stagnation temperature), and with the diffusive reflection of molecules from the surface. The model of hard sphere molecules was used. The calculations were carried out by DSMC, which details were described in [7]. For the comparison of the numerical solution by DSMC method with macroscopic theories four

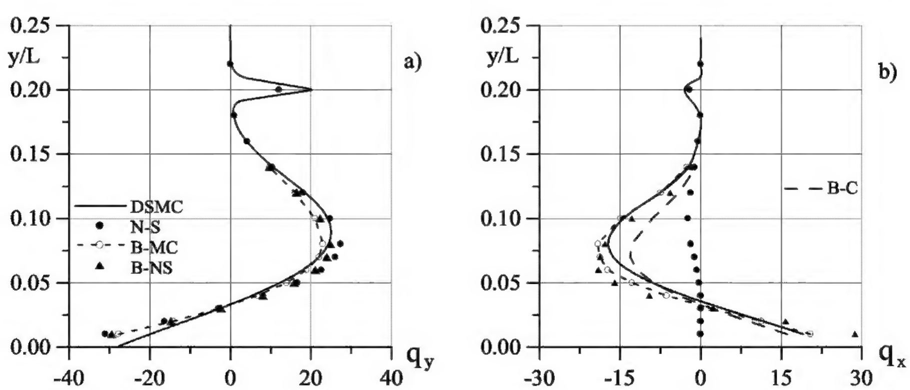

FIGURE 4. Normal (a) and tangential (b) heat fluxes vs. coordinate y for $\mathrm{Re}_{\infty,x} = 8000$: DSMC result and different approximations.

approximations were used: Navier-Stokes approximation (designation in figures is N-S), full Burnett approximation with its terms being described as usual through p_{ij}^N and q_i^F (B-NS), full Burnett approximation with its terms

expressed through numerical results p_{xy} and q_y (B-MC), and approximation defined by relations (1) derived for the Couette flow (B-C).

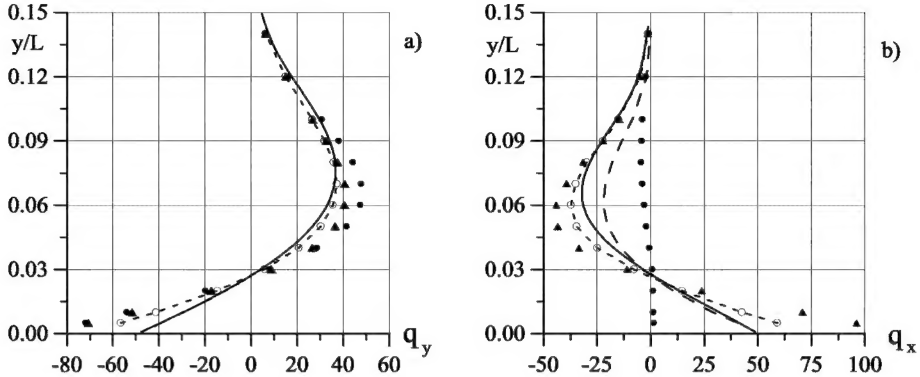

FIGURE 5. Normal (a) and tangential (b) heat fluxes vs. coordinate y for $\mathrm{Re}_{\infty,x} = 4200$: DSMC result and different approximations. Designations are so as in Fig. 4.

Figure 4 shows that at sufficiently small Kn number ($\mathrm{Re}_{\infty,x} = 8000$) the profile of the heat flux normal to the plate can be well defined by macroscopic relations - both by Navier-Stokes relations (Fourier relation) and full Burnett relations. The tangential heat flux cannot be defined in Navier-Stokes approximation, but three variants of Burnett approximation give its approximately correct quantitative description. At larger Kn numbers ($\mathrm{Re}_{\infty,x} = 4200$, Fig. 5) any Burnett approximation can qualitatively define profiles of heat flux only. In this case the approximations similar to the ones given in Fig. 1 should be derived.

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REFERENCES

1. Nosik,V.I., "Degeneration of the Chapman-Enskog expansion in one-dimensional motion of Maxwellian molecule gases" in *Rarefied Gas Dynamics*, edited by O.M.Belotserkovsky et al., Proc.13th Symp., Plenum Press, New York-London, 1985, pp. 237-244.
2. Brey, J.J., Santos, A., *Phys. Rev.*, **36**, N.6, 2842-2849 (1987).
3. Lee, C.J., *AIAA J.*, **32**, N.5, 985-990 (1994).
4. Zhong, X., Koura, K., "Comparison of solutions of the Burnett equations, Navier-Stokes equations, and DSMC for Couette flow" in *Rarefied Gas Dynamics*, edited by J.Harvey and G.Lord, Proc.19th Symp., Oxford Univ. Press, New York, 1995, pp. 354-360.
5. Galkin, V.S., Shavaliev, M.Sh., *Fluid Dynamics*, **33**, N.4, 469-487 (1998).
6. Bishaev, A.M., Rykov,V.A., *Fluid Dynamics*, **15**, N.3, (1998).
7. Erofeev, A.I., Yegorov I.V., "Comparison of Navier-Stokes and Monte-Carlo solutions for a wide range of similarity parameters" in *Rarefied Gas Dynamics*, ed. Ching Shen, Proc.20th Symp., Peking Univ. Press, Beijing, China, 1987, pp. 321-326.